Central Place Indexing: Optimal Location Representation for Digital Earth

Kevin M. Sahr
Department of Computer Science
Southern Oregon University

Southern
Terra
Cognita
Laboratory

The Situation

- Geospatial computing has achieved many impressive results
- But it now faces unprecedented challenges

Advent of “Digital Earth”

- exemplifies the challenges facing geospatial computing, combining in one platform:
  - ✦ “mother of all (geospatial) databases”
  - ✦ simulation, interactive 3D visualization, & analysis of vast quantities of diverse distributed global geospatial “big data”
  - ✦ integrates real-time location update and manipulation

The Key: Location Representation

- to implement this vision in totality a revolution in our fundamental approaches to geospatial computing is required
- at the core of all geospatial applications are data structures that represent location
    - ✦ even minor efficiency improvements in location representation can lead to substantial performance increases

Fundamental Location Representation Types

- digital earth systems must provide data structures for representing:
    - ✦ raster/pixels for
        - ✤ imagery
        - ✤ discrete simulation
        - ✤ “gridded” data analysis
    - ✦ vector/point locations
    - ✦ spatial databases/spatial indexes

Traditional Raster Location Representation

- raster of square pixels
- addressed using 2-tuple of integers

Traditional Image Processing Model

- traditional raster representation supports image processing based on a conceptual model of:
  - ✦ input from square raster of sensors
  - ✦ stored internally as matrix of pixels
  - ✦ displayed one-to-one on a square raster of display pixels

Digital Earth Reality

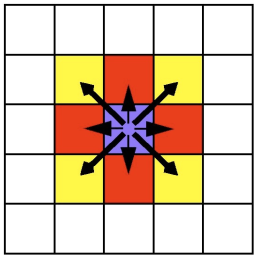
- image processing in digital earth systems breaks this mold
  - processed satellite image pixels rarely correspond to individual sensors
  - must support whole-earth image sets
    - spherical topology
  - internal pixels mapped to virtual 3D surface for display

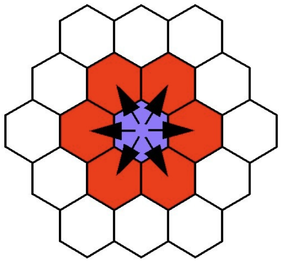
A Superior Alternative

- numerous researchers have proposed using hexagon-shaped pixels, arranged in a hexagonal raster
- the human eye uses a hexagonal arrangement of photoreceptors
- compared to square rasters, hexagon rasters
  - ✦ are 13% more efficient at sampling
  - ✦ 25% to 50% more efficient for common image processing algorithms

Discrete Simulation

- hexagonal grids also have numerous advantages over square grids for discrete simulation
  - ✦ superior angular resolution
  - ✦ discrete distance metric better approximates cartesian distance
  - ✦ display uniform unambiguous adjacency





Traditional Vector Location Representation

- 3- or 2-tuples of floating point values
- attempt to mimic the real number coordinates used in pre-computer scientific analysis and 2D mapping

But…

- vectors of real numbers
  - ✦ are continuous and infinite
- tuples of floating point values
  - ✦ are discrete and finite

Problems

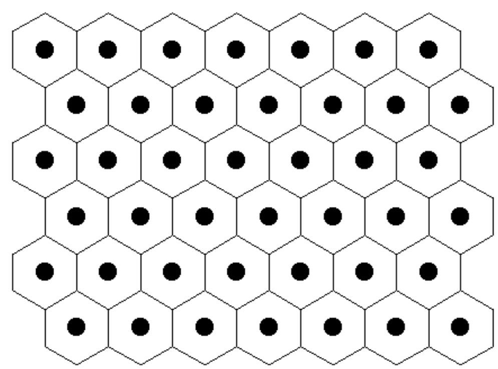
- the simplest operations (e.g., equality test) can result in profound semantic errors
- bounding the rounding error on individual operations can be difficult
  - ✦ on complex systems can be impossible

The Reality

- floating point values are no more “exact” than integer values
- given ***n*** bits, we can distinguish 2^n unique values
  - ✦ all other points must be quantized to these
- ***all*** computer representations of location — both raster ***and*** vector — are necessarily discrete

A Superior Alternative

- the human brain represents location using a hexagonal arrangement of neurons
- quantization to the points of a hexagonal lattice is optimal using multiple formulations
  - ✦ least average quantization error
  - ✦ covering problem
  - ✦ packing problem



Traditional Spatial DBs

- traditional raster and vector representations are inefficient for many common spatial operations
- spatial DBs add a linear spatial index
  - correspond to buckets containing locations, providing
    - more efficient spatial queries
    - coarse filter for specific algorithms

Traditional Spatial DBs

- underlying vector/raster representation retained for
  - ✦ final stage of many algorithms
  - ✦ arbitrary spatial operations
- form of spatial DBs based on traditional vector/raster representational forms
  - ✦ structured: square quad tree
  - ✦ semi-structured: rectangular buckets (e.g. R-tree)

Spatial Queries

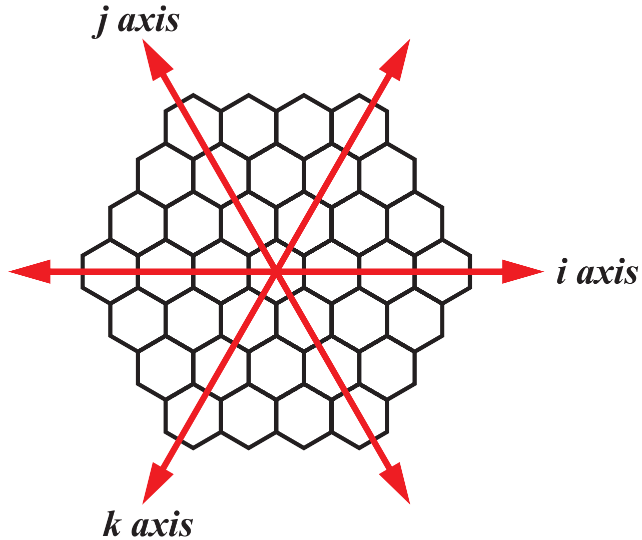
- traditional primary spatial query type: window/axes-aligned rectangle
- ***but*** primary query type in modern geospatial systems is proximity
  - ✦ recall that hexagonal discrete distance metric better approximates cartesian distance
    - ✤ hexagon buckets provide more efficient proximity coarse filter

The Task

- design a hierarchical integer index for hexagon lattices that can be used for:
  - ✦ multi-precision vector location
  - ✦ multi-resolution raster location
  - ✦ structured spatial index
- must be explicitly spherical
- Digital Earth Primary Spatial Index:
  - ✦ One Index to Rule Them All

Hexagon Coordinate Systems

- single resolution hexagon grids have three natural axes spaced at 120° angles

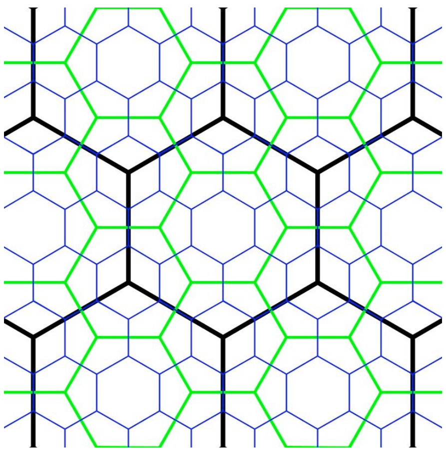


Hexagonal Multi-Res

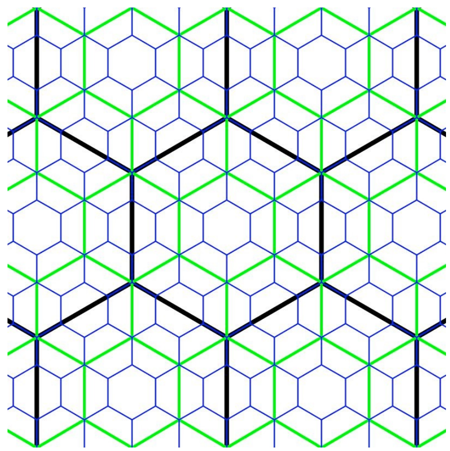
- regular multi-precision/resolution hexagon lattices can be created with an infinite number of apertures
    - ✦ aperture: ratio of cell areas between resolutions
- research has focused on the Central Place (Christaller, 1966) apertures 3, 4, and 7

Central Place Apertures

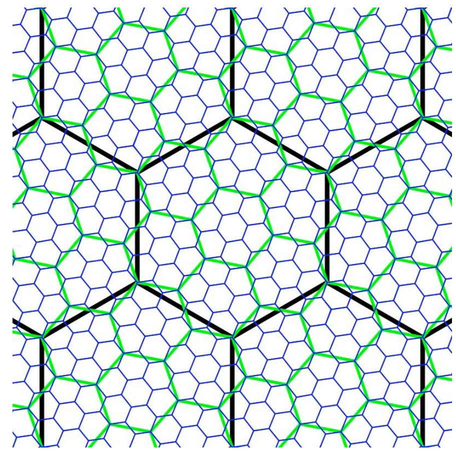
aperture 3



aperture 4



aperture 7

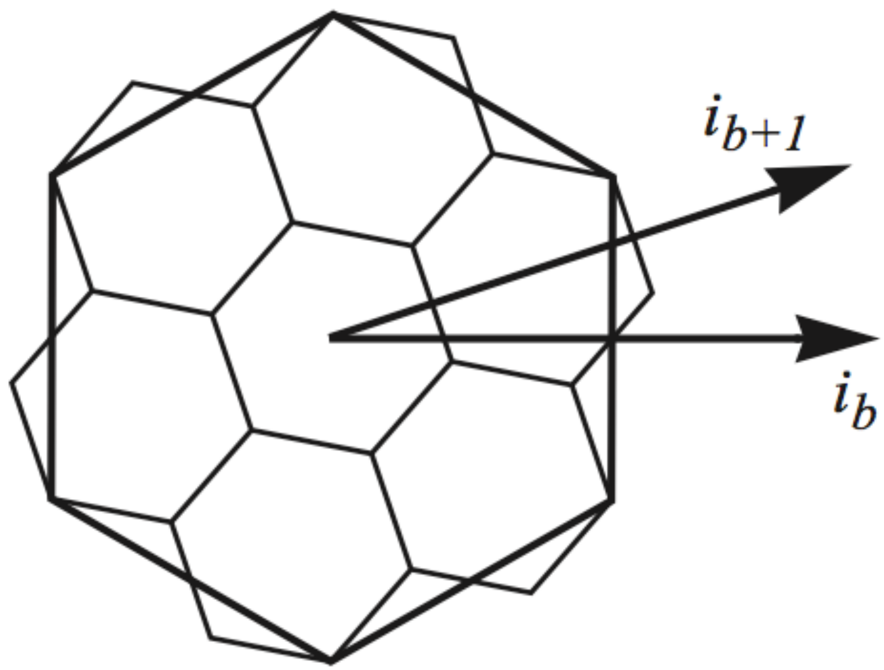


Prefix Codes

- hierarchical prefix codes have many advantages for hierarchical spatial indexes
  - each digit in index corresponds to a single precision in the representation
    - provides locality preserving total ordering
    - implicitly encodes precision
    - provides efficient generalization via truncation

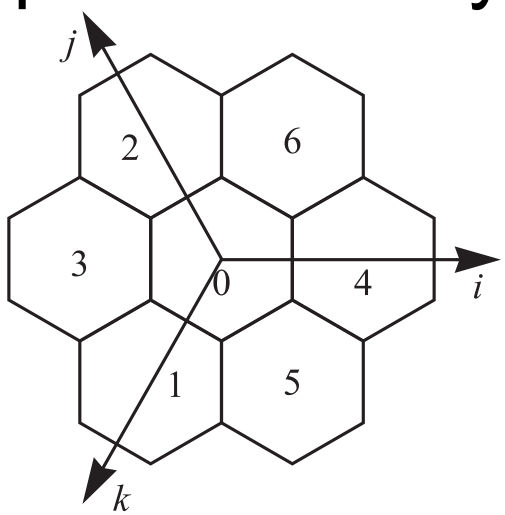
Aperture 7 Case

- note that each hexagon is naturally associated with 7 hexagons at the next finer resolution



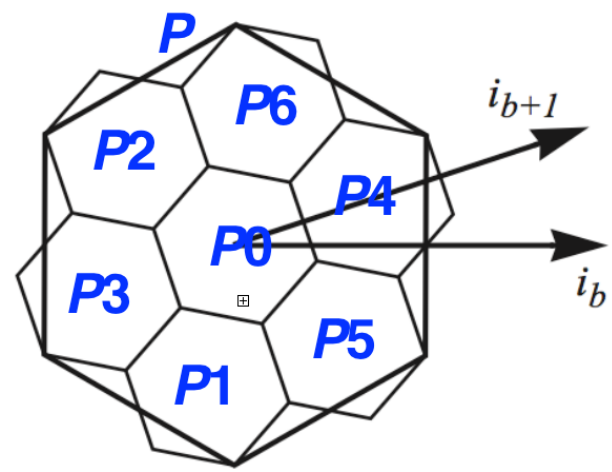
GBT

- Generalized Balanced Ternary (GBT) (Gibson & Lucas, 1982) is a hierarchical prefix code system for aperture 7 grids



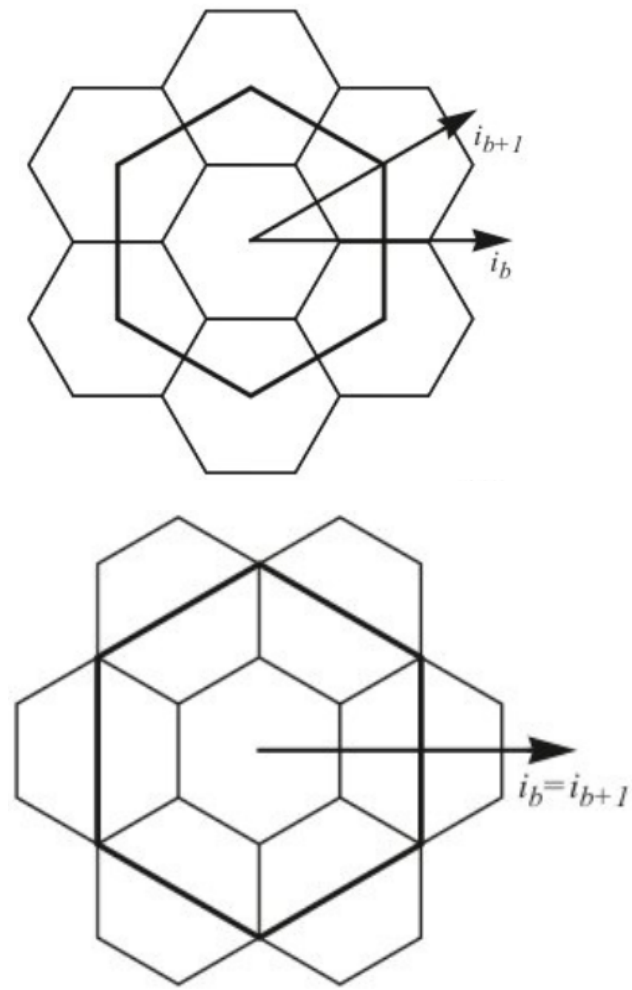
- single digits correspond to each possible hexagonal direction

- each indexing child adds the appropriate digit to their parent's index



Apertures 3 and 4

- note that in apertures 3 and 4 each cell also naturally has 7 finer precision potential indexing children

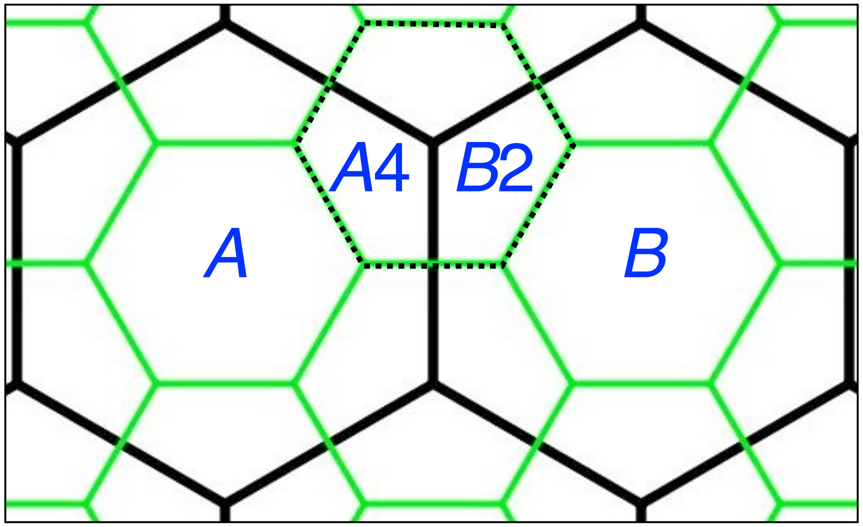


Central Place Indexing

- we can apply the GBT arrangement to the aperture 3 and 4 cases
- we call the result Central Place Indexing (CPI)
  - ✦ provides uniform indexing for all 3 apertures
  - ✦ allows for indexing mixed-aperture resolution sequences

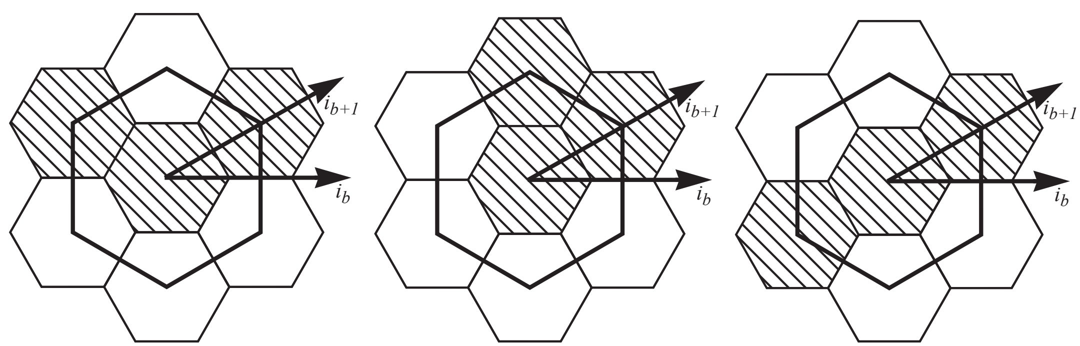
Pixel/Bucket Indexing

- cells in aperture 3 and 4 resolutions can have multiple parents cells and therefore multiple valid CPI indexes
  - ✦ aperture 3 example:



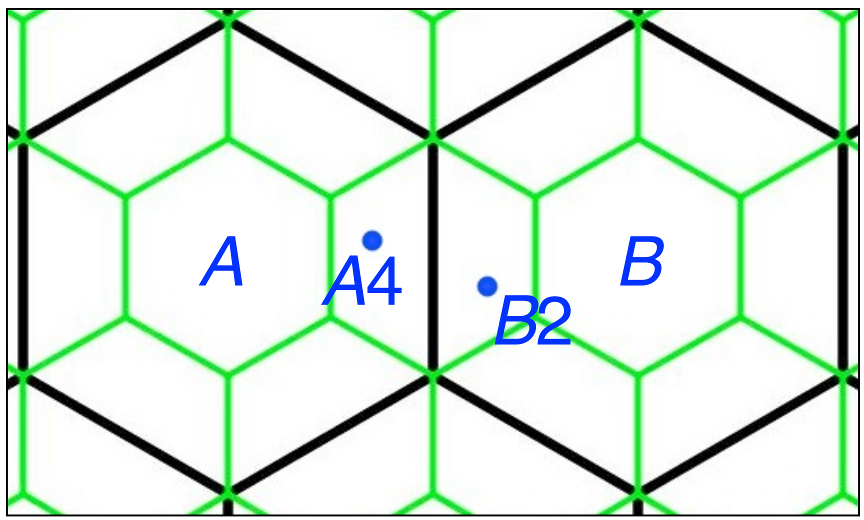
Pixel/Bucket Indexing

- if the cells represent pixels or DB buckets, then a single unique index must be chosen for each cell
  - ✦ a consistent choice of child assignment must be made
  - ✦ example aperture 3 solutions:



Vector Indexing

- in apertures 3 and 4 point quantization can be performed at each successive resolution



Vector Indexing

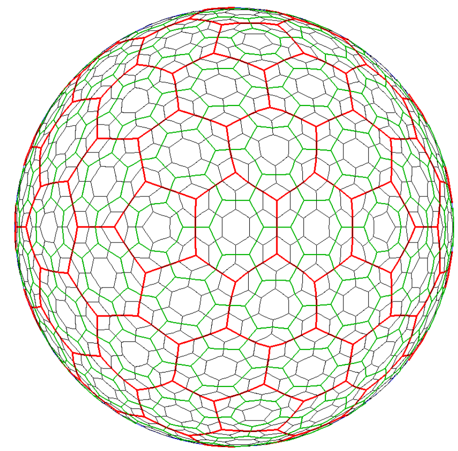
- thus aperture 3 and 4 grids effectively address cell sub-regions
  - ✦ provides true multi-precision point quantization
  - ✦ truncation and rounding are equivalent
  - ✦ indexes can be progressively transmitted

CPI Algorithms

- we have implemented planar CPI algorithms for
  - forward & inverse quantization
  - addition/translation
  - subtraction
  - metric distance
- implemented using efficient per-digit table lookups

The Sphere

- we can apply any CPI system to a spherical icosahedron to index a hexagonal Discrete Global Grid System (DGGS)



- note that cells centered on the icosahedral vertices are pentagons
  - ✦ we can apply CPI indexing to them by deleting one of the non-centroid indexing sub-sequences

Conclusions

- multi-resolution hexagonal DGGSs provide the best known basis for raster, vector, and spatial DB location representation for digital earth systems
- CPI provides a unified efficient hierarchical indexing for all types of location representation on hexagonal DGGSs

SOUTHERN
TERRA
COGNITA
LABORATORY

www.discreteglobalgrids.org

2