# The Superfund_500m Discrete Global Grid

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## Introduction

The **Superfund_500m** grid was commissioned by the US Environmental Protection agency for use in developing its Superfund Emergency Response Atlas. The grid is a hierarchically indexed icosahedral hexagonal discrete global grid (DGG) (Sahr et al. 2003) consisting of approximately 22 hectare hexagons, with approximately 500 meter distance between hexagon centers. This cell size is generated geometrically by creating a mixed aperture sequence of two aperture 4 subdivisions followed by 15 aperture 3 subdivisions.

It is not possible to create a network of equal area, equal shape, and equally spaced grid cells greater than twenty in number on the surface of a sphere. Thus one or more, often two or all three, of these characteristics are distorted to varying degrees across the surface. The approach used here starts with the twenty triangular faces of the icosahedron, creates a regular, equal area, equal shape, and equally spaced network of hexagons of the desired size on one or more of the planar triangles and then projects these cells to the surface of the globe. The distortion characteristics of this approach have been investigated by Kimerling et al. (1999) and Gregory et al. (2008).

Grid characteristics for the 10 addressable **Superfund_500m** resolutions are given in Table 1.

## Grid Construction

The cells are generated as regular hexagons (and pentagons) on the surface of an icosahedron, oriented relative to the globe so as to be symmetrical about the equator. The cells are projected to longitude and latitude on a sphere with the authalic WGS84 radius (NAD 83 datum) using the inverse icosahedral projection of R. Buckminster Fuller (1975) as developed analytically by Robert Gray (1995) and John Crider (2008).

The **Superfund_500m** cell identifiers are an instance of Central Place Indexing (CPI). CPI (Sahr, 2011; Sahr, submitted 2018) is a class of hierarchical indexing systems for pure and mixed aperture hexagonal DGGs, where the linear index assigned to each cell is constructed as a path address (Sahr 2008) on a multi-resolution discrete global grid system with the specified aperture sequence. A CPI addressing system was used in the initial design for a sampling system for the EPA's Environmental Monitoring and Assessment Program (White et al. 1992).

**Table 1.** Superfund_500m DGG Resolutions (see notes after table).

| Resolution | # Cells[1] | Hex Area[2] (sq. km) | Intercell Distance[3] (km) | CLS[4] (km) |
|---|---|---|---|---|
| 0 | 42 | 12,751,640.5431 | 3,526.8262 | 4,046.3596 |
| 1 | 162 | 3,187,910.1358 | 1,763.4131 | 2,016.7939 |
| 2 | 1,442 | 354,212.2373 | 587.8044 | 671.6409 |
| 3 | 12,962 | 39,356.9153 | 195.9348 | 223.8573 |
| 4 | 116,642 | 4,372.9906 | 65.3116 | 74.6182 |
| 5 | 1,049,762 | 485.8878 | 21.7705 | 24.8727 |
| 6 | 9,447,842 | 53.9875 | 7.2568 | 8.2909 |
| 7 | 85,030,562 | 5.9986 | 2.4189 | 2.7636 |
| 8 | 765,275,042 | 0.6665 | 0.8063 | 0.9212 |
| 9 | 2,295,825,122 | 0.2222 | 0.4655 | 0.5319 |

**Table 1 Notes:**
[1]At every resolution 12 of the cells are pentagons and the remainder are hexagons.
[2]The 12 pentagons have an area exactly 5/6 the area of a hexagon.
[3]Measured in the plane of the Fuller projection space.
[4]Characteristic Length Scale (CLS): the diameter of a spherical cap of the same area as a cell of the specified resolution. This metric was suggested by Ralph Kahn.

The **Superfund_500m** CPI system was designed to meet two design goals. First, the CPI approach allows the grid to have an intercell spacing of approximately 500 meters, which cannot be achieved with sufficient accuracy using a pure aperture grid system. Second, in order to take advantage of the pre-existing discrete global grid software tool **DGGRID** the cells needed to be hierarchically indexed in such a manner that the Christaller sets of each base cell are each restricted to a single **ij** coordinate system whose axes form two of the edges of a spherical quadrilateral formed by a pair of adjacent icosahedral faces.

These design goals were met by constructing a grid with base cells of valence 5 (i.e., with pentagonal voronoi areas) centered on each of the 12 vertices of an icosahedron and then applying the following aperture sequence:

$$4, 4, 3^{ccw}, 3^{cw}, 3^{ccw}, 3^{cw}, 3^{ccw}, 3^{cw}, 3^{ccw}, 3^{cw}, 3^{ccw}, 3^{cw}, 3^{ccw}, 3^{cw}, 3^{ccw}, 3^{cw}, 3^{ccw}$$

To assign a unique hierarchical index to each cell, as well as to achieve the remaining grid design goals, the generator types ***A-K*** were defined with the following generator string representations:

***A***: ***A***123456
***B***: ***C***123***CCC***
***C***: ***D***123***EED***
***D***: ***F***123***IK***6
***E***: ***J***123***GH***6
***F***: ***D***123***EE***6
***G***: ***DD***234***E***6
***H***: ***EE***2***D***456

***I***: ***E***1***DD***456
***J***: ***E***1***D***345***E***
***K***: ***D***123***E***5***D***

Two of the base cells on opposing sides of the icosahedron are assigned generator type ***A***, while the remaining 10 base cells (centered on the remaining 10 icosahedral vertices) are assigned generator type ***B***. Figures 1-3 illustrate the resolution 17 regions corresponding to the resolution 1 grid cells, numbered 10-51 (to avoid leading zeros in indexes). The resolution 0 base tiles centered on the resolution 1 cells labeled 10 and 51 are the two base cells that were assigned generator type ***A***, which generates a single pentagonal cell at all resolutions.

**Index Form**

The base tiles of the **Superfund_500m** CPI indexes correspond to resolution 1 DGG cells, as illustrated in Figure 1. Base tiles 11-50 each have four hierarchical children at the aperture 4 resolution 2, which are assigned the additional digits 1-4. These resolution 2 cells each have three children at resolution 3, which are assigned the additional digits 1-3; this assignment continues recursively through the aperture 3 resolutions 3-17. The special base tiles 10 and 51 have a single hierarchical child at all resolutions, which is assigned the additional digit 1 at each resolution. In order to reduce the length of indexes, a first order compression is performed by grouping the aperture 3 resolution 3-16 digits into pairs and replacing each pair of digits with a digit value of 1-9.

The assignment of digits has been chosen so that all indexes form integers with the same number of digits, and so that possibly troublesome leading zeros can be avoided, even if redundant leading digits are removed from indexes when working in regions where this is possible.

Thus the full resolution 17 **Superfund_500m** CPI indexes are condensed into indexes with 11 digits, resulting in 10 addressable grid resolutions (see Table 1).

A **Superfund_500m** index at full resolution (resolution 9) has the 11-digit form:

*BB499999993*

where:

- *BB* is the 2-digit resolution 1 base tile cell with values 10-51,
- *4* is the resolution 2 aperture 4 digit with values 1-4,
- each *9* represent two successive combined aperture 3 digits for resolutions 3-16, with values 1-9 each, and
- the final 3 represents the extra aperture 3 at resolution 17, with values 1-3

Note that the resolution 9 footprint of base tiles 11-50 form approximate spherical diamonds on the surface of the globe, as illustrated in Figure 1. Figures 2 and 3 illustrate the resolution 9 indexing footprints of base tiles adjacent to base tiles 10 and 15 respectively. The hierarchical children at each aperture 3 resolution form a compact triangle; Figure 4 illustrates the pattern formed at resolution 9.

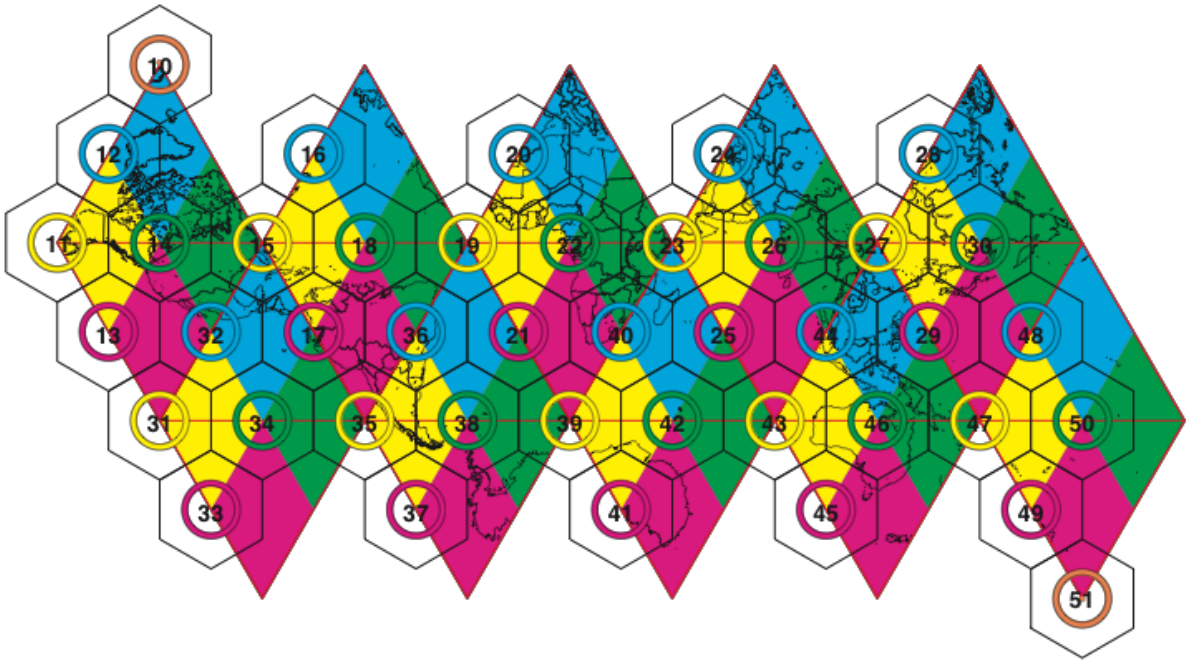


**Figure 1. Superfund_500m** base tiles on an unfolded icosahedron with corresponding resolution 9 indexing footprints. Cells within the footprint region of a base tile will have that base tile's index as the first two digits of the cell's index. Tiles 10 and 51 index only a single cell at all resolutions, centered on the corresponding base tile. Note that all tiles that are centered on a triangle vertex are actually pentagons on the sphere.

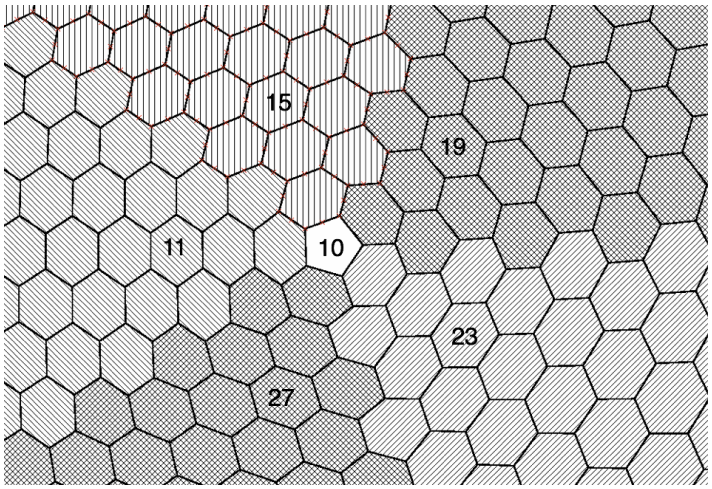


**Figure 2.** Region around base tile 10 at resolution 9.

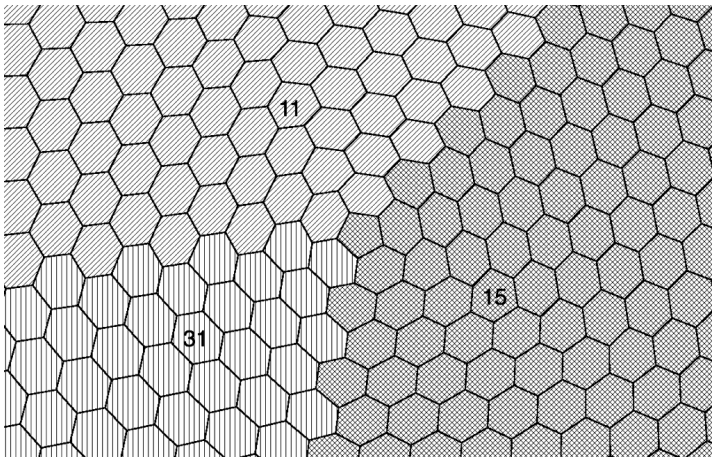


**Figure 3.** Region around base tile 15 at resolution 9.

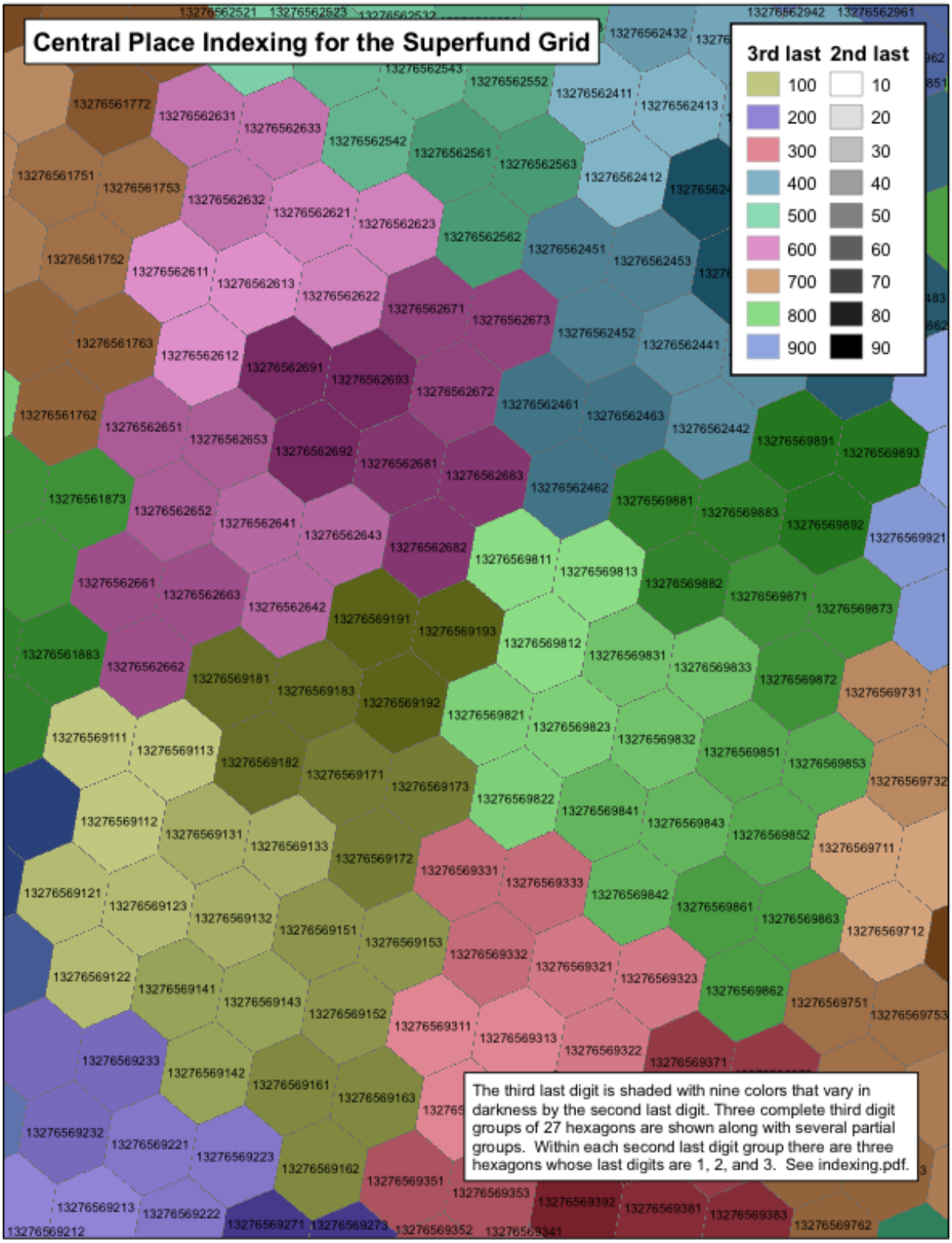


**Figure 4. Superfund_500m** resolution 9 tiling pattern.

## Appendix F. References


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